UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS **9709/11**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2013**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

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READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

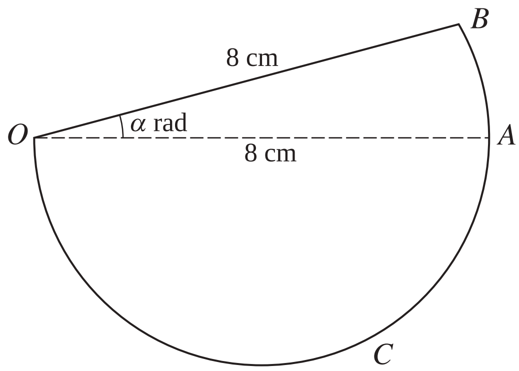
JC13 06_9709_11/2R

1 It is given that $f(x) = (2x - 5)^3 + x$, for $x \in \mathbb{R}$. Show that f is an increasing function. [3]

2 **(i)** In the expression $(1 - px)^6$, p is a non-zero constant. Find the first three terms when $(1 - px)^6$ is expanded in ascending powers of x. [2]

(ii) It is given that the coefficient of x^2 in the expansion of $(1 - x)(1 - px)^6$ is zero. Find the value of p. [3]

3



In the diagram, OAB is a sector of a circle with centre O and radius 8 cm. Angle BOA is α radians. OAC is a semicircle with diameter OA. The area of the semicircle OAC is twice the area of the sector OAB.

(i) Find α in terms of π. [3]

(ii) Find the perimeter of the complete figure in terms of π. [2]

4 The third term of a geometric progression is -108 and the sixth term is 32. Find

(i) the common ratio, [3]

(ii) the first term, [1]

(iii) the sum to infinity. [2]

5 **(i)** Show that $\dfrac{\sin\theta}{\sin\theta + \cos\theta} + \dfrac{\cos\theta}{\sin\theta - \cos\theta} \equiv \dfrac{1}{\sin^2\theta - \cos^2\theta}$. [3]

(ii) Hence solve the equation $\dfrac{\sin\theta}{\sin\theta + \cos\theta} + \dfrac{\cos\theta}{\sin\theta - \cos\theta} = 3$, for $0° \leqslant \theta \leqslant 360°$. [4]

6 Relative to an origin O, the position vectors of three points, A, B and C, are given by

$$\overrightarrow{OA} = \mathbf{i} + 2p\mathbf{j} + q\mathbf{k}, \quad \overrightarrow{OB} = q\mathbf{j} - 2p\mathbf{k} \quad \text{and} \quad \overrightarrow{OC} = -(4p^2 + q^2)\mathbf{i} + 2p\mathbf{j} + q\mathbf{k},$$

where p and q are constants.

(i) Show that $\overrightarrow{OA}$ is perpendicular to $\overrightarrow{OC}$ for all non-zero values of p and q. [2]

(ii) Find the magnitude of $\overrightarrow{CA}$ in terms of p and q. [2]

(iii) For the case where $p = 3$ and $q = 2$, find the unit vector parallel to $\overrightarrow{BA}$. [3]

7 A curve has equation $y = x^2 - 4x + 4$ and a line has equation $y = mx$, where m is a constant.

(i) For the case where $m = 1$, the curve and the line intersect at the points A and B. Find the coordinates of the mid-point of AB. [4]

(ii) Find the non-zero value of m for which the line is a tangent to the curve, and find the coordinates of the point where the tangent touches the curve. [5]

8 **(i)** Express $2x^2 - 12x + 13$ in the form $a(x + b)^2 + c$, where a, b and c are constants. [3]

(ii) The function f is defined by $f(x) = 2x^2 - 12x + 13$ for $x \geqslant k$, where k is a constant. It is given that f is a one-one function. State the smallest possible value of k. [1]

The value of k is now given to be 7.

(iii) Find the range of f. [1]

(iv) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1}. [5]

9 A curve has equation $y = f(x)$ and is such that $f'(x) = 3x^{\frac{1}{2}} + 3x^{-\frac{1}{2}} - 10$.

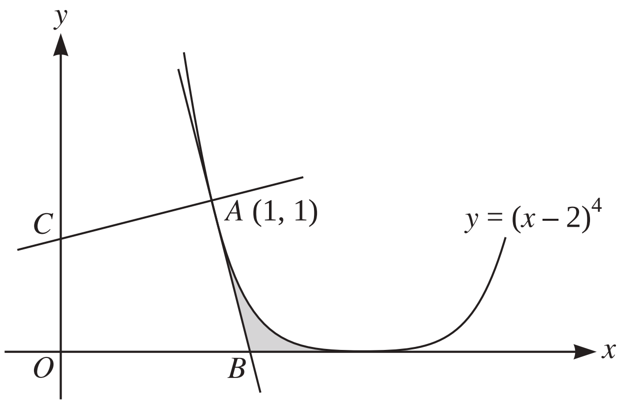
(i) By using the substitution $u = x^{\frac{1}{2}}$, or otherwise, find the values of x for which the curve $y = f(x)$ has stationary points. [4]

(ii) Find $f''(x)$ and hence, or otherwise, determine the nature of each stationary point. [3]

(iii) It is given that the curve $y = f(x)$ passes through the point $(4, -7)$. Find $f(x)$. [4]

[Question 10 is printed on the next page.]

10



The diagram shows part of the curve $y = (x-2)^4$ and the point $A\,(1,\ 1)$ on the curve. The tangent at A cuts the x-axis at B and the normal at A cuts the y-axis at C.

(i) Find the coordinates of B and C. [6]

(ii) Find the distance AC, giving your answer in the form $\dfrac{\sqrt{a}}{b}$, where a and b are integers. [2]

(iii) Find the area of the shaded region. [4]